Form 4 Chapter 2. Quadratic Equations

1. Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$ax^2 + bx + c = 0$$
$$x^2 + \frac{b}{a}x + \frac{c}{a} = 0$$
$$x^2 + \frac{b}{a}x = -\frac{c}{a}$$
$$x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 = -\frac{c}{a} + \left(\frac{b}{2a}\right)^2$$
$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}$$
$$x = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

2. Forming quadratic equation

$$\boxed{x^2 - \{SOR\}x + \{POR\} = 0}$$

Proof:

$$x = \alpha, x = \beta$$
$$x - \alpha = 0, x - \beta = 0$$
$$(x - \alpha)(x - \beta) = 0$$
$$x^2 - (\alpha + \beta)x + \alpha\beta = 0$$
$$x^2 - \{SOR\}x + \{POR\} = 0$$

3. SOR/ POR

By comparing $x^2 - (\alpha + \beta)x + \alpha\beta = 0$ with the general form of a quadratic equation $ax^2 + bx + c = 0$

$$\div a,\ x^2 + \frac{b}{a}x + \frac{c}{a} = 0$$

$\therefore$ Sum of Roots, $\boxed{SOR:\ \alpha + \beta = -\frac{b}{a}}$, Product of Roots, $\boxed{POR:\ \alpha\beta = \frac{c}{a}}$

Form 4 Chapter 4. Indices, Surds and Logarithms

4. Multiplication Law of Logarithms

$x = a^p \quad \Rightarrow \quad p = \log_a x$

$y = a^q \quad \Rightarrow \quad q = \log_a y$

$xy = a^p \times a^q$

$\quad = a^{p+q}$

$\Rightarrow \log_a xy = p + q$

$$\boxed{\log_a xy = \log_a x + \log_a y}$$

5. Division Law of Logarithms

$x = a^p \quad \Rightarrow \quad p = \log_a x$

$y = a^q \quad \Rightarrow \quad q = \log_a y$

$\frac{x}{y} = a^p \div a^q$

$\quad = a^{p-q}$

$\Rightarrow \log_a \frac{x}{y} = p - q$

$$\boxed{\log_a \frac{x}{y} = \log_a x - \log_a y}$$

6. Power Law of Logarithms

$x = a^p \quad \Rightarrow \quad p = \log_a x$

$y = a^q \quad \Rightarrow \quad q = \log_a y$

$x^m = \left(a^p\right)^m$

$\quad = a^{pm}$

$\Rightarrow \log_a x^m = pm$

$$\boxed{\log_a x^m = m \log_a x}$$

7. Changing The Base Of Logarithms

Let $\log_a b = m \quad \Rightarrow \quad a^m = b$

Taking $\log_c$ on both sides, $\log_c a^m = \log_c b$

$m \log_c a = \log_c b$

$$\Rightarrow \boxed{\log_a b = \frac{\log_c b}{\log_c a}}$$

If $c = b \quad \Rightarrow \quad \log_a b = \frac{\log_b b}{\log_b a}$

$$\boxed{\log_a b = \frac{1}{\log_b a}}$$

Form 4 Chapter 5. Progressions

8. General Formula for an Arithmetic Progression

$T_1 = a \qquad T_1 = a + (1-1)d$

$T_2 = a + d \qquad T_2 = a + (2-1)d$

$T_3 = a + 2d \qquad T_3 = a + (3-1)d$

$T_4 = a + 3d \qquad T_4 = a + (4-1)d$

$\therefore \ T_n = a + (n-1)d$

9. Sum of the first n^{th} terms of an Arithmetic Progression

Let the arithmetic progression be: $a, a+d, a+2d, \ldots, a+(n-1)d$

Sum of its terms, $S_n = a + a + d + a + 2d + \ldots + a + (n-2)d + a + (n-1)d \ \cdots (1)$

Rewriting backwards, $S_n = a + (n-1)d + a + (n-2)d + \ldots + a + d + a \ \cdots (2)$

$(1)+(2) \quad 2S_n = n\left[2a + (n-1)d\right]$

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

10. Formula for General Term of a Geometric Progression

$T_1 = a \qquad T_1 = ar^{1-1}$

$T_2 = ar \qquad T_2 = ar^{2-1}$

$T_3 = ar^2 \qquad T_3 = ar^{3-1}$

$T_4 = ar^3 \qquad T_4 = ar^{4-1}$

$\therefore \ T_n = ar^{n-1}$

11. Sum of the first n^{th} terms of a Geometric Progression

$$S_n = a + ar + ar^2 + \ldots + ar^{n-2} + ar^{n-1} \cdots (1)$$

$$(1)\times r, \quad rS_n = ar + ar^2 + ar^3 + \ldots + ar^{n-1} + ar^n \cdots (2)$$

$(2)-(1), \quad rS_n - S_n = ar^n - a \qquad\qquad (1)-(2), \quad S_n - rS_n = a - ar^n$

$(r-1)S_n = ar^n - a \qquad\qquad (1-r)S_n = a - ar^n$

$S_n = \dfrac{a(r^n - 1)}{r-1} \quad r > 1 \qquad\qquad S_n = \dfrac{a(1-r^n)}{1-r} \quad r < 1$

12. Sum to infinity of a Geometric Progression

From $S_n = \dfrac{a(1-r^n)}{1-r}$,

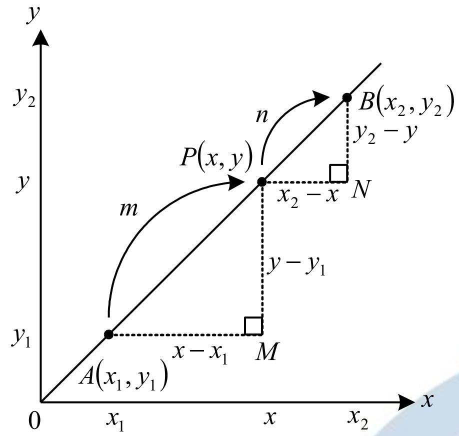
for the case of $-1 < r < 1$ and n approaches infinity, $n \to \infty$, the value of $r^n \cong 0$

$$S_\infty \cong \frac{a(1-0)}{1-r}$$

Hence, $S_\infty = \dfrac{a}{1-r}$

Form 4 Chapter 7. Coordinate Geometry

13.Divisor of a Line Segment



If $\dfrac{AP}{PB} = \dfrac{m}{n}$, and ΔAPM and ΔPBN are similar,

$$\Rightarrow \frac{AM}{PN} = \frac{AP}{PB} \quad \text{and} \quad \frac{PM}{BN} = \frac{AP}{PB}$$

$$\frac{x - x_1}{x_2 - x} = \frac{m}{n} \qquad \frac{y - y_1}{y_2 - y} = \frac{m}{n}$$

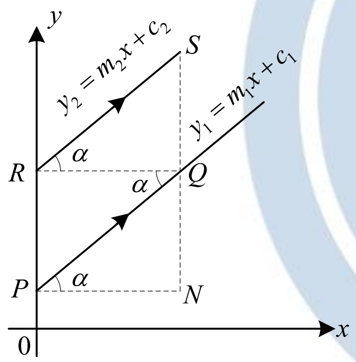
$$nx - nx_1 = mx_2 - mx \qquad ny - ny_1 = my_2 - my$$

$$(m+n)x = nx_1 + mx_2 \qquad (m+n)y = ny_1 + my_2$$

$$x = \frac{nx_1 + mx_2}{m+n} \qquad y = \frac{ny_1 + my_2}{m+n}$$

$$(x, y) = \left(\frac{nx_1 + mx_2}{m+n}, \frac{ny_1 + my_2}{m+n} \right)$$

14. Parallel Lines



In the given diagram, line PQ is parallel to line RS and line RQ and PN are parallel to the x-axis

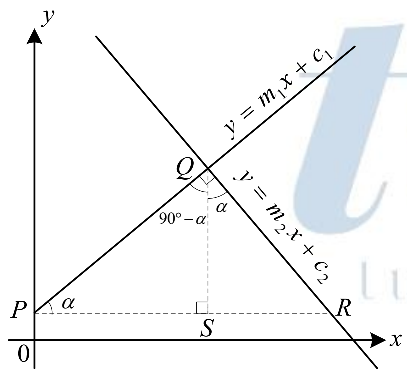
$\angle NPQ = \angle PQR = \angle QRS = \alpha$ (Alternate angles)

Gradient PQ, $m_1 = \dfrac{QN}{PN} = \tan \alpha$

Gradient RS, $m_2 = \dfrac{SQ}{RQ} = \tan \alpha$

Hence, $m_1 = m_2$

15. Perpendicular Lines



- In the given diagram, if PQ is perpendicular to QR, $\angle PQR = \angle RQS = \alpha$

Gradient PQ, $m_1 = \dfrac{QS}{PS}$

$= \tan \alpha$

Gradient QR, $m_2 = -\dfrac{QS}{SR}$

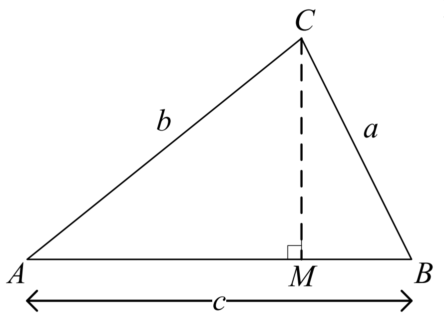
$= -\dfrac{1}{\tan \alpha}$

$$\therefore m_1 \cdot m_2 = (\tan \ \alpha) \cdot \left(-\frac{1}{\tan \ \alpha} \right)$$

$$m_1 \cdot m_2 = -1$$

Form 4 Chapter 9. Solution of Triangle

16. Sine Rule



From ΔACM, $\sin A = \dfrac{CM}{b}$

$\Rightarrow CM = b\sin A \cdots (1)$

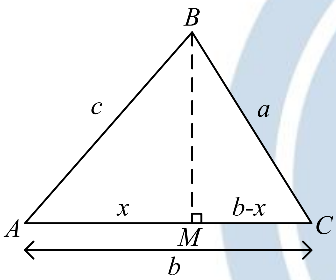
From ΔBCM, $\sin B = \dfrac{CM}{a}$

$\Rightarrow CM = a\sin B \cdots (2)$

$(1) = (2)$ $\quad b\sin A = a\sin B$

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

17. Cosine Rule



Let $AM = x$ $MC = b - x$.

From ΔABM, $BM^2 = c^2 - x^2 \cdots (1)$

$\cos A = \dfrac{x}{c}$

$\Rightarrow x = c\cos A \cdots (2)$

From ΔCBM, $BM^2 = a^2 - (b-x)^2$

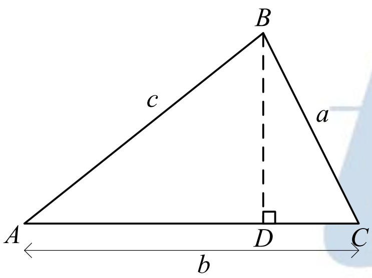
$= a^2 - b^2 + 2bx - x^2 \cdots (3)$

Comparing $(1) \& (3)$, $c^2 - x^2 = a^2 - b^2 + 2bx - x^2$

$a^2 = b^2 + c^2 - 2bx$

Hence $a^2 = b^2 + c^2 - 2bc\cos A$ [using (2)]

18. Area Of Triangle



In ΔABC, line BD is perpendicular to line AC.

In ΔBDC, $\sin C = \dfrac{BD}{a}$

$\Rightarrow BD = a\sin C \cdots (1)$

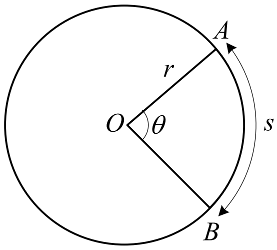
Area of $\Delta ABC = \dfrac{1}{2} \times b \times BD \cdots (2)$

$= \dfrac{1}{2} b(a\sin C)$

$= \dfrac{1}{2} ab\sin C$

Form 5. Chapter 1. Circular Measure

19. Length Of Arc

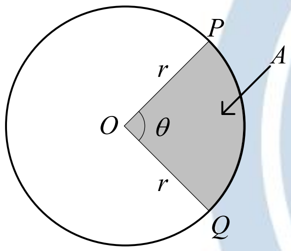


$$\frac{\text{length of arc } AB}{\text{length of circumference of a circle}} = \frac{\text{angle } AOB}{\text{total angles of a circle}}$$

$$\frac{s}{2\pi r} = \frac{\theta}{2\pi}$$

$$s = r\theta$$

20. Area Of Sector Of A Circle



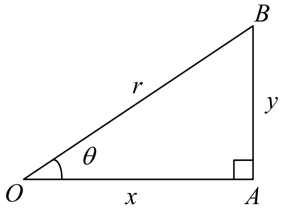
$$\frac{\text{area of sector}}{\text{area of circle}} = \frac{\text{angle } POQ}{\text{total angles of a circle}}$$

$$\frac{A}{\pi r^2} = \frac{\theta}{2\pi}$$

$$A = \frac{1}{2}r^2\theta$$

Form 5. Chapter 6. Trigonometric Functions

21. Basic Trigonometric Identities



$\sin\theta = \dfrac{y}{r}$, $\cos\theta = \dfrac{x}{r}$

From Pythagoras Theorem,

$$OA^2 + AB^2 = OB^2$$

$$x^2 + y^2 = r^2$$

$$\div r^2,\quad \left(\frac{x}{r}\right)^2 + \left(\frac{y}{r}\right)^2 = 1$$

$$\cos^2\theta + \sin^2\theta = 1 \cdots (1)$$

$$(1) \div \cos^2\theta,\quad 1 + \tan^2\theta = \sec^2\theta$$

$$(1) \div \sin^2\theta,\quad \cot^2\theta + 1 = \operatorname{cosec}^2\theta$$

22. Double Angle Formulae

For $\sin 2A$, $\sin(A + A) = \sin A\cos A + \cos A\sin A$,

$$\boxed{\sin 2A = 2\sin A \cos A}$$

- For $\cos 2A$, $\cos(A + A) = \cos A\cos A - \sin A\sin A$

$$\boxed{\cos 2A = \cos^2 A - \sin^2 A} \quad \text{or} \quad \cos 2A = \cos^2 A - (1 - \cos^2 A) \quad \text{or} \quad \cos 2A = (1 - \sin^2 A) - \sin^2 A$$

$$\boxed{\cos 2A = 2\cos^2 A - 1} \qquad \boxed{\cos 2A = 1 - 2\sin^2 A}$$

- For $\tan 2A$, $\tan(A + A) = \dfrac{\tan A + \tan A}{1 - \tan A\tan A}$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$